Trigonométrie hyperbolique

Définition

$$\forall x \in \mathbb{R} \qquad \operatorname{sh} x = \frac{e^x - e^{-x}}{2} \qquad \operatorname{ch} x = \frac{e^x + e^{-x}}{2} \qquad \operatorname{th} x = \frac{\operatorname{sh} x}{\operatorname{ch} x}$$

Formule fondamentale : $\operatorname{ch}^2 - \operatorname{sh}^2 = 1$.

Formules d'addition et de soustraction (hors programme)

$$\operatorname{ch}(a+b) = \operatorname{ch} a \operatorname{ch} b + \operatorname{sh} a \operatorname{sh} b \qquad \operatorname{ch}(a-b) = \operatorname{ch} a \operatorname{ch} b - \operatorname{sh} a \operatorname{sh} b$$

$$\operatorname{sh}(a+b) = \operatorname{sh} a \operatorname{ch} b + \operatorname{ch} a \operatorname{sh} b \qquad \operatorname{sh}(a-b) = \operatorname{sh} a \operatorname{ch} b - \operatorname{ch} a \operatorname{sh} b$$

$$\operatorname{th}(a+b) = \frac{\operatorname{th} a + \operatorname{th} b}{1 + \operatorname{th} a \operatorname{th} b} \qquad \operatorname{th}(a-b) = \frac{\operatorname{th} a - \operatorname{th} b}{1 - \operatorname{th} a \operatorname{th} b}$$

Formules de duplication (hors programme)

$$\operatorname{ch} 2a = \operatorname{ch}^2 a + \operatorname{sh}^2 a = 2\operatorname{ch}^2 a - 1 = 2\operatorname{sh}^2 a + 1$$

$$\operatorname{sh} 2a = 2 \operatorname{sh} a \operatorname{ch} a$$

$$\operatorname{th} 2a = \frac{2 \operatorname{th} a}{1 + \operatorname{th}^2 a}$$

Parité

Les fonctions sh et th sont impaires. La fonction ch est paire.

Dérivation

$$\operatorname{sh}' = \operatorname{ch} \qquad \operatorname{ch}' = \operatorname{sh} \qquad \operatorname{th}' = 1 - \operatorname{th}^2 = \frac{1}{\operatorname{ch}^2}$$

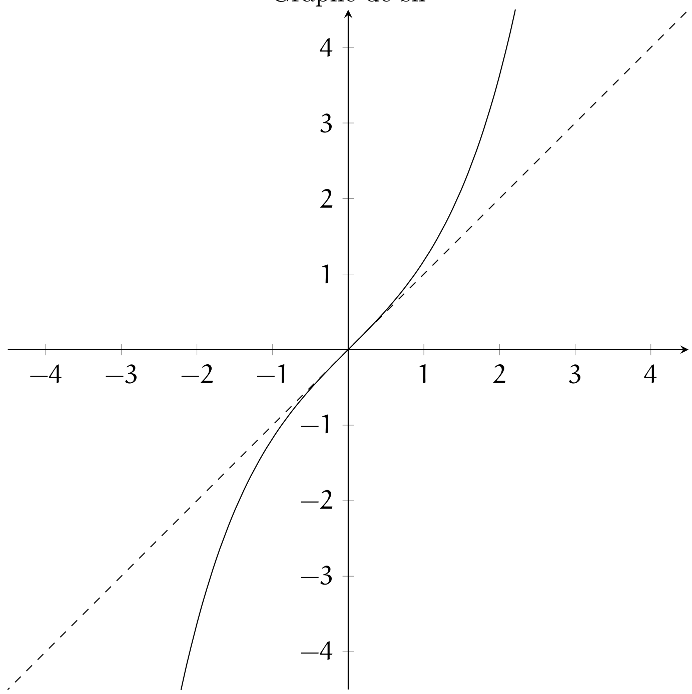
Limites

$$\lim_{x \to +\infty} \operatorname{sh} x = +\infty \qquad \lim_{x \to +\infty} \operatorname{ch} x = +\infty \qquad \lim_{x \to +\infty} \operatorname{th} x = 1$$

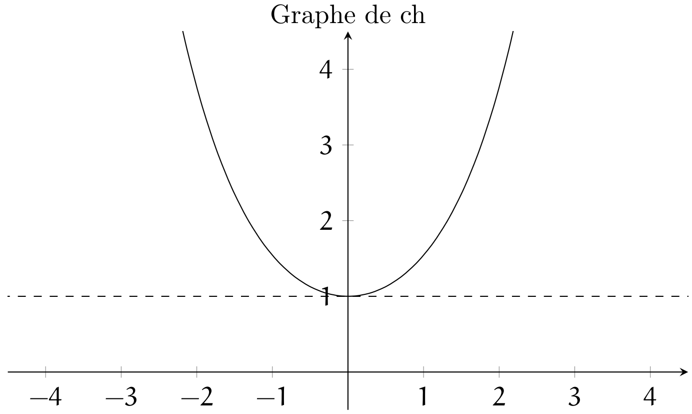
$$\lim_{x \to -\infty} \operatorname{sh} x = -\infty \qquad \lim_{x \to -\infty} \operatorname{ch} x = +\infty \qquad \lim_{x \to -\infty} \operatorname{th} x = -1$$

Graphes

Graphe de sh



Graphe de ch



Graphe de th