TRIGONOMÉTRIE

Formules d'addition et de soustraction

$$\cos(a + b) = \cos a \cos b - \sin a \sin b \qquad \sin(a + b) = \sin a \cos b + \cos a \sin b \qquad \tan(a + b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

$$\cos(a - b) = \cos a \cos b + \sin a \sin b \qquad \sin(a - b) = \sin a \cos b - \cos a \sin b \qquad \tan(a - b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

Cas particuliers

$$\cos\left(x + \frac{\pi}{2}\right) = -\sin x \qquad\qquad \sin\left(x + \frac{\pi}{2}\right) = \cos x \qquad\qquad \tan\left(x + \frac{\pi}{2}\right) = -\frac{1}{\tan x}$$

$$\cos\left(x - \frac{\pi}{2}\right) = \sin x \qquad\qquad \sin\left(x - \frac{\pi}{2}\right) = -\cos x \qquad\qquad \tan\left(x - \frac{\pi}{2}\right) = -\frac{1}{\tan x}$$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x \qquad\qquad \sin\left(\frac{\pi}{2} - x\right) = \cos x \qquad\qquad \tan\left(\frac{\pi}{2} - x\right) = \frac{1}{\tan x}$$

$$\cos(x + n\pi) = (-1)^n \cos x \qquad\qquad \sin(x + n\pi) = (-1)^n \sin x$$

Formules de duplication

$$\cos 2a = \cos^2 a - \sin^2 a \qquad\qquad\qquad \sin 2a = 2 \sin a \cos a \qquad\qquad \tan 2a = \frac{2 \tan a}{1 - \tan^2 a}$$

$$= 2 \cos^2 a - 1 = 1 - 2 \sin^2 a$$

$$\cos^2 a = \frac{1 + \cos 2a}{2} \qquad\qquad\qquad\qquad \sin^2 a = \frac{1 - \cos 2a}{2}$$

Formules de factorisation

$$\cos a + \cos b = 2 \cos \frac{a + b}{2} \cos \frac{a - b}{2} \qquad\qquad \sin a + \sin b = 2 \sin \frac{a + b}{2} \cos \frac{a - b}{2}$$

$$\cos a - \cos b = -2 \sin \frac{a + b}{2} \sin \frac{a - b}{2} \qquad\qquad \sin a - \sin b = 2 \cos \frac{a + b}{2} \sin \frac{a - b}{2}$$

Formules de linéarisation

$$\cos a \cos b = \frac{1}{2} \left(\cos(a + b) + \cos(a - b)\right) \qquad\qquad \sin a \sin b = \frac{1}{2} \left(\cos(a - b) - \cos(a + b)\right)$$

$$\sin a \cos b = \frac{1}{2} \left(\sin(a + b) + \sin(a - b)\right) \qquad\qquad \cos a \sin b = \frac{1}{2} \left(\sin(a + b) - \sin(a - b)\right)$$
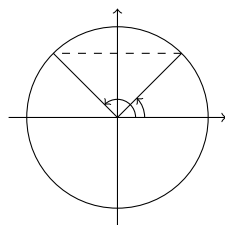
Paramétrage rationnel du cercle trigonométrique

$$\cos \theta = \frac{1 - t^2}{1 + t^2} \qquad\qquad \sin \theta = \frac{2t}{1 + t^2} \qquad\qquad \tan \theta = \frac{2t}{1 - t^2} \qquad\qquad \left(\text{avec } t = \tan \frac{\theta}{2}\right)$$
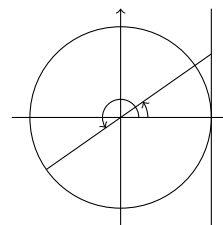
Equations trigonométriques
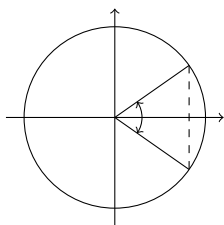
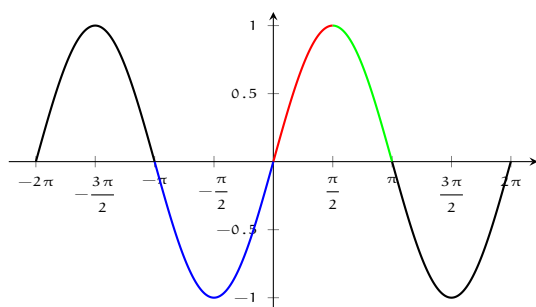$$\cos a = \cos b \Leftrightarrow \begin{cases} a \equiv b[2\pi] \\ \text{ou} \\ a \equiv -b[2\pi] \end{cases} \qquad \sin a = \sin b \Leftrightarrow \begin{cases} a \equiv b[2\pi] \\ \text{ou} \\ a \equiv \pi - b[2\pi] \end{cases} \qquad \tan a = \tan b \Leftrightarrow a \equiv b[\pi]$$
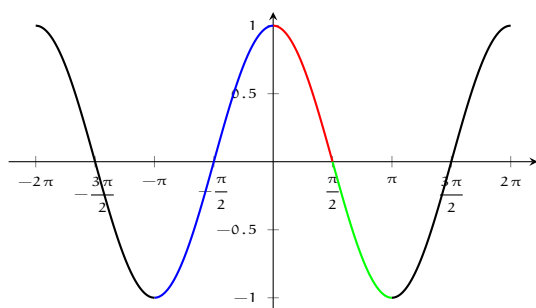
Graphes

Graphe de sin
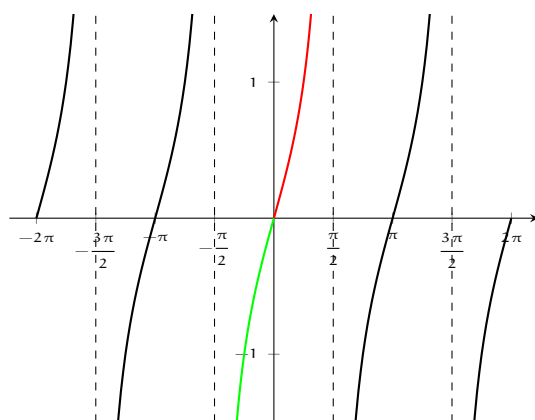
- Etude sur $\left[0, \frac{\pi}{2}\right]$
- Prolongement par symétrie d'axe $x = \frac{\pi}{2}$
- Prolongement par parité
- Prolongement par 2π-périodicité

Graphe de cos

- Etude sur $\left[0, \frac{\pi}{2}\right]$
- Prolongement par symétrie de centre $\left(\frac{\pi}{2}, 0\right)$
- Prolongement par parité
- Prolongement par 2π-périodicité

Graphe de tan

- Etude sur $\left[0, \frac{\pi}{2}\right]$
- Prolongement par parité
- Prolongement par π-périodicité